This problem is for the numerical analysis of the error in the approximation of a function using blobs in 1 dimension. Given the data \((x_k, f_k)\) for \(k = 1, 2, \ldots, N\) for a function defined on the interval \([a, b]\), we can construct a blob approximation to the function \(f(x)\) by a midpoint rule approximation of the convolution \((f \star \phi_\delta)(x)\):

\[
f(x) \approx \sum_{k=1}^{N} f_k h \phi_\delta(x - x_k)
\]

where \(h = (b - a)/N\) and \(x_k = a + (k - 1/2)h\), \(f_k = f(x_k)\).

Consider the function \(f(x) = 1 + \cos(x)\) for \(-\pi \leq x \leq \pi\).

1) Choose a second-order blob.

2) Fix \(N\) and generate the data \((x_k, f_k)\).

3) Find the approximation (1) for several values of \(\delta\) (larger and smaller than \(h\)) and plot the graph of the error as a function of \(\delta\).

4) Let \(N\) be twice as large and do it again.

5) Do this for enough values of \(h\) so that you can draw some conclusions about how the error depends on both \(h\) and \(\delta\). Try to find a relationship between the optimal value of \(\delta\) and \(h\).

6) Repeat the entire exercise with a fourth-order blob.