1. Look up the help on the MATLAB command \texttt{spline} and read about how to use it. What type of end-point conditions are possible with MATLAB? (look for not-a-knot in the textbook to find out what it is). Suppose you must fit a spline through the data

\[
\begin{array}{|c|c|c|c|c|}
\hline
x & 0 & 2 & 4 & 5 & 9 \\
\hline
y & 1.0000 & 2.5713 & 6.3579 & 8.5544 & 18.7217 \\
\hline
\end{array}
\]

and suppose the end-point conditions are \( f'(0) = 1 \) and \( f'(9) = -1 \). How would you find the spline using MATLAB? Explain in full detail.

2. In class we discussed cubic splines with \textit{natural} \((S_0''(x_0) = 0, S_{n-1}''(x_n) = 0)\) or \textit{clamped} \((S_0'(x_0) = \alpha, S_{n-1}'(x_n) = \beta)\) end-point conditions. The matrices to compute the coefficients \(c_k\) were identical except for the first and last rows. Write down what the first and last rows are for these two cases. Write down the equations for \textit{periodic} boundary conditions in terms of \(S_0\) and \(S_{n-1}\) and write down the first and last rows of the matrix for \textit{periodic} boundary conditions? Explain.

3. Suppose you have 5 data points \((x_0, y_0), (x_1, y_1), \ldots, (x_4, y_4)\) that come from the function \(y = x^3\). Suppose you use a natural cubic spline to fit the data, will you capture the exact function or will there be an error in the approximation? Explain your answer.

4. Copy from the class website the MATLAB function \texttt{MySpline.m}. Add comments to the file describing exactly what each of the lines does.

5. Use \texttt{MySpline.m} to find splines for \(x\) and \(y\) in parametric form (you have to define a parameter). Plot the spline for \(y\) vs. the spline for \(x\) together with the data points.

\[
\begin{array}{|c|c|c|c|}
\hline
x & 0.50 & 0.05 & 0.42 & -1.26 \\
\hline
y & -1.50 & -1.30 & 0.55 & 1.00 \\
\hline
\end{array}
\]