THE MATHEMATICAL SEEDS OF ECONOMIC COLLAPSE

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Abstract. We point out how the over confidence in mathematical theories for analyzing derivatives and the breakdown in real communication between academics and financial analysts has lead to the current world financial collapse.

1. INTRODUCTION

The idea that credit default swaps, derivatives, and high leverage buying are at the foundation of the current world wide financial collapse actually misses the true cause. It is as if our financial leaders are looking right at the forest but only seeing the trees. Much of leverage buying is based on economic theories of how to predict the evolution of the financial world. Principal among these theories is the theory of derivative pricing which gave financial leaders of the latter part of the twentieth century the mistaken impression that the financial world was virtually as predictable as a statistical mechanical system in physics. Consequently, they virtually turned over their decision making to a cadre of highly trained mathematical technicians who were told to merely compute the results of these theories. The result in effect is like a ship with a blind person at the tiller being directed by a captain from below deck. Of course the result is eventually disaster. One should realize this is at the heart of Alan Greenspan's recent statement, "I found a flaw in the model of the critical fundamental structure of the way the world works." It seems clear at this point that those in charge have not properly realized where their mistake is and what mathematics must be developed before the mistake can have even a ghost of a chance of being fixed.

2. DERIVATIVE PRICING

We have all heard that the main culprit in the current world wide financial crisis is the toxic effect of credit default swaps which are spread throughout the financial world accelerating the collapse. Of course, a large part of the problem can be attributed to leverage buying on the part of large financial institutions, as well as the purchase of derivatives. In effect, a derivative is itself a vehicle of leverage. Moreover, a credit default swap is really just another example of a derivative. Thus, the problem underlying the mess is the incorrect theory of derivative pricing and the overconfidence placed in it in recent history for shaping the outlook of financial leaders as to the confidence that can be placed in computerized calculations of outcomes.

To properly understand how the theory of derivative pricing could be at the basis of our mess, one must understand a little bit about how it works. The details are the subject of many textbooks by now, but the theory at its fundamental level, mathematically goes back to Albert Einstein [2] who developed it to analyze Brownian motion. As a theory of statistical physics, it correctly lead the world's physicists to believe in the molecular theory of matter, which prior to that time had been held by most physicists and chemists as merely a convenient way to think about the world for solving certain problems and not as a realistic picture of the matter in the universe. What Einstein realized was that if molecules were real and if matter consisted of molecules, then the motion of these molecules could be used to predict aspects of the motion of tiny particles that were bumped by the molecules. Thus, when you observe the motion of tiny lint particles in a beam of sunlight, you are seeing the effect of molecules of air bumping them. What Einstein did was to develop a statistical theory which could predict how

far a lint particle could move as it was being repeatedly bumped by air molecules. Actually he was thinking of small pollen particles suspended in a liquid, after Robert Brown who had observed the motions microscopically much earlier. But the theory is the same in either case. The mathematics was then developed into the theory of stochastic processes and stochastic differential equations. Then, in the latter part of the twentieth century, economists began to think of the large aggregate results of competitive buying and selling as having a similar effect on prices in analogy with the random bumps of molecules on macroscopic particles. If so, then the theory of stochastic differential equations could be applied to calculate outcomes in certain economic situations and this lead to the theory of option pricing and the derivative market. At its foundation then is the belief that the overall market forces on prices can be viewed as the result of little "bumps" to prices caused by individual purchases and sales of small parcels of stock. It turned out that an especially convenient equation could be formulated using an overall assumed inflation rate as a sort of overall drift in prices, together with an assumed amount of volatility in prices. The resulting formula due to the economists Black, Merton and Scholes, [1], for which they won the Nobel prize in economics, has been widely used by financial institutions for pricing options of all kinds. The mathematics underlying Einstein's analysis of Brownian motion has lead to the development of an entire field of mathematics, and there is no doubt that the mathematics underlying the Black-Scholes-Merton Formula is correct. To get an idea of the mathematics involved, the interested reader can track back from some of the latest developments as found in [3] and [4]. It leads through some of the major developments of twentieth century mathematics. Keeping in mind the imprimatur that the Black-Merton-Scholes equation received from its creators having received a Nobel prize in economics, one begins to understand how otherwise sober financial leaders might be lead to try to apply such an equation to option pricing. The results seemed to work spectacularly, so as long as the market performed well, the hedge funds could use this to make big profits which seemed to be virtually guaranteed. The financial leaders felt they had finally cracked the secret of market prediction. Just hire some mathematical technocrats who can computerize all the calculations and let the good times roll. Of course, there were a few minor scares along the way, but nothing that seemed to call into question the mathematical models being applied. And of course, the applied mathematicians worked like sorcerer's apprentices, never questioning whether the underlying foundational assumptions of the models are actually true. They were just following the orders of their masters to use the accepted formulas. On the other hand, their masters were accepting the methods as having all the certainty of mathematics, since of course, a Nobel prize in economics had resulted.

In my opinion, this situation resulted in an army of technocrats inventing ever more complicated derivative vehicles, as the technocrats did not ever view questioning the basic foundational assumptions in real terms as part of their job. They did not connect any calculations with actual human behavior. As far as they were concerned, people were just like molecules. On the other hand, as long as the mathematical methods produced spectacular results, the financial leaders who should have been more suspicious, forgot that in applying any mathematical theory to the real world one must seriously examine the basic assumptions on which the theory rests. Again, there was a Nobel prize. It is as if an army of mathematical surveyors of the universe used Euclidean geometry without question, without ever asking a physicist if the geometry of the universe is actually Euclidean (in fact the universe has curvature so is definitely not Euclidean). So, the basic assumption needs to be examined. Are investors like molecules? If they were, there would rarely be a run on a bank, because it would be as if one gas molecule could actually perceive what the other gas molecules are doing and try to anticipate the results of what the other molecules do so as to gain some kind of advantage. Something is clearly wrong here, and obviously the molecular model of the financial world begins to seriously break down when investors begin trying to anticipate what all the other investors are doing in situations where it becomes obvious what everyone else wants to do, as in the case of bank runs. In normal times

investors simply try to figure the value of an investment in some intrinsic terms as opposed to trying to synchronize their investing with the herd.

3. SYNCHRONICITY

Unfortunately, the word synchronicity has been used by a host of different popular intellectual and pseudo-intellectual writers to refer to unexplained coincidences. A sort of New Age phenomena. On the other hand, the mathematical analysis of the emergence of synchronized behavior in the natural world has been very difficult, and until recently has been almost a complete mathematical mystery. Here we are referring not to mere coincidences but the emergence of actual order out of seeming chaos. For instance, in Russia audiences after performances typically engage in synchronized clapping, without need of any leader to beat a common time, and mathematicians observing this noticed that the synchronization happens much more rapidly than one would suspect in ordinary familiar statistical terms. People start clapping randomly at first, and then in a few seconds, it all becomes synchronized. There are also many examples in the insect world of synchronized behavior, and it is only recently that mathematicians have begun to understand how to model the interactions that lead to its emergence. The interested reader can consult [5] to get an idea of the complications here (see [6] for a readable review of this book). What is important for financial analysis is that the phenomena of bank runs and stock market crashes seem to be of the same nature: spontaneously emerging synchronized behavior. The market is going along perfectly well, and then all of a sudden the investors all start moving in synchrony trying to do the same thing. If they are all buying, it of course makes all the investors happy who got in on the ground floor, whereas, in the case they all start selling in sync, the end result can be disaster. Roughly speaking, the mathematical models here are indicating that the interaction of participants in spontaneously erupting synchronization are in fact violating the molecular assumptions of the Black-Scholes-Merton equation. This indicates that the molecular assumptions of the Black-Scholes-Merton equation are only approximately correct in stable market environments, and that a correct model would actually be violently different when the market starts to become unstable. In effect, the models used by financial technocrats have been leading us all off of a cliff when a correct model-if one could be developed would predict when spontaneous order in the market would emerge and thereby give sufficient advance warning. When markets are in the unstable regime, our current models are navigating blindly toward the economic shoals. My own feeling on this is that the primitive state of the mathematics in this area will not lead to applicable results in the near future, as far as financial analysis is concerned.

4. THE FUTURE OF FINANCIAL ANALYSIS

Since the mathematics of financial analysis cannot be depended upon to give reliable results when we need them most, our financial leaders must realize that they must go on their real financial experience of the world and not rely on mathematical technocrats to tell them what to do. It might be many years before reliable mathematical models can be developed that can replace the Black-Merton-Scholes equation in application to financial prediction. In the mean time, we must realize that the mathematical technocrats must be highly regulated with some financial common sense so as to avoid financial disasters. We cannot turn our financial futures over to some mindless computers.

5. THE BLAME

It is tempting for liberals to want to blame conservatives for the predicament we find ourselves in as a lot of the trouble can be traced back to deregulation. In fact, it was natural for economic conservatives to be more taken in by the notion that the market behaves like some kind of statistical machine with complete predictability. It sort of justifies the Adam Smith view of the "invisible hand". However, liberal economists were clearly also taken in as the Clinton

administration and Senator Dod, one of the leaders of the Senate banking committee, signed on to many of the moves toward deregulation. When one is presented with a mathematical model that one does not understand but which seems to work and which has the academic world and Nobel prizes behind it, then its application seems to trump political orientation. Given the apparent predictability of markets as in option pricing, it could seem very reasonable to drop margin requirements from 10 percent down to 2.5 percent, no matter your political point of view. The real fault lies in allowing greed to trump good sense on all sides. When something is too good to be true it probably is. When economists present us with theories that seem to predict the financial future, we need to examine the underlying assumptions with extreme caution, paying close attention to where violations of underlying assumptions might be. Can we blame the mathematical technocrats? Certainly not. They were merely following the dictates of financial leaders who they thought had certified the basic underlying assumptions to be true. Moreover, the salaries paid such technocrats far surpassed academic salaries (typically by a factor of ten, even though still a mere pittance in comparison to their masters' incomes), so there was little motivation to question. What we have is sort of a perfect storm of organizational dysfunction in the way our financial world is controlled.

6. THE WAY OUT

Since our mathematical models cannot be depended upon to behave reliably in our current financial condition, we must proceed with extreme caution. In particular, computerized hedge fund trading and option pricing must be severely regulated from now on. We should all realize that we are all at fault for our current state of affairs, and consequently, we must organize and develop an equitable means of sharing the financial pain. Bailing out the big financial powers so as to save the billionaires is neither equitable nor useful. The Federal Reserve which controls the money supply can use computers wisely to remove many of the middle men from the process of supplying money and credit to the business community in order to keep business going. Consumers and workers need not fear if the wheels of business can keep going. For the Federal Reserve to give out more money to banks which are still afraid to loan out the money to businesses that need it, will not solve any problems in the long run. In fact, there is no reason why a mechanism cannot be set in motion whereby homeowners and small businesses can borrow money directly from the Federal Reserve at the Federal Reserves' low interest rate, using computers. This would stimulate the economy, remove many of the useless banking middle men, and as well provide competition among bankers which would lead to more lending and lower interest rates. It would allow the little guy on Main Street to enjoy some of the advantages previously only available to the big guys on Wall Street.

In the long run, we have to face the fact that as a nation we are tremendously under-educated when it comes to technical matters of any kind. And we face an ever more complex world in every way. We must spend more effort to properly educate our people so as to give them the ability to make the best decisions in complex situations. A useful tool in this direction would be the requirement of full disclosure in all financial matters. If person A purchases something from person B, person A should have complete information as to what person B stands to gain from the sale. We have developed an economy which largely fuels the rich by running on the ignorance of the population as a whole. Where expertise must be counted on, effective regulations must be enacted to protect the consumer from fraudulent banking practices which were caused by deregulations allowed by the republican congresses of the last fifteen years and which have run riot in the first decade of the current century.

As far as education is concerned, much more attention must be paid to practical matters of living in our complex world. It is no wonder that typical inner city high school students pay no attention in class when almost nothing of real usefulness is taught. Every student must be taught how to grow basic vegetables on a small plot of land, how to prepare a decent meal, how to fix a car, how to fix basic problems around the home, and perform basic plumbing repair and small home remodelling jobs. In addition, each student must learn reading and basic

mathematics, so as to be able to find basic information from books in libraries or online, and make proper economic decisions. Each student should accomplish this by the end of the eighth grade, after which the student should be prepared to enter the work world or go on to high school. High school should be restricted to those students who want to learn and make an effort to succeed. It may be that more students will learn literature and poetry if we restrict its availability than if we try to force it down their throats. In any case, there is little evidence that information can be efficiently forced into peoples' brains. By the time a student finishes high school the student should have mastered mathematics through calculus as well as a course in chemistry and a course in physics. It is also time to realize that other countries are better than we are in many ways, especially as regards education and medical care. We need to look around and learn from them.

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