Your Instructor: _____ Your Name: _____

MATH 121 FINAL EXAMINATION

Monday, December 14, 1998, 6:00 P.M.-10:00 P.M.

10 points

1) Check the following statements

	True	**False**
a. $f(x) = \sin(\lvert \sin x \rvert)$ is an even function.	☐	☐
b. The equation $x = \cos x$ has exactly one solution.	☐	☐
c. There are functions which are differentiable but not continuous.	☐	☐
d. The function $f(x) = \lvert x^2 - 1 \rvert$ has more than two critical points.	☐	☐
e. The acceleration function is always an antiderivative of the velocity function.	☐	☐
f. The function $f(x) = \lvert x \rvert$ is differentiable at $x = 0$.	☐	☐
g. $f(x) = x^{1/3}$ has an inflection point with a vertical tangent line at $x = 0$.	☐	☐
h. If $f'(a) = 0$, then f has a local minimum or maximum at $x = a$.	☐	☐
i. Since $\tan \pi = 0$ while $\tan(\frac{\pi}{4}) = 1$, the function $f(x) = \tan x$ does not increase on every interval in its domain.	☐	☐
j. If $F(x) = \int_0^x \sin(t^2)\, dt$, then $F(x) \leq x$ for all $x \geq 0$.	☐	☐

5 points

2) Consider the function f and check the appropriate box.

$$f(x) = \begin{cases} |x+2| \ , & x < -2 \\ (x+2)^2 \ , & -2 \leq x < -1 \\ 2x+3 \ , & -1 \leq x < 0 \\ x^2 \ , & 0 \leq x < 1 \\ 2x \ , & 1 \leq x < 2 \\ 4 \ , & 2 \leq x \end{cases}$$

12 points

3) Suppose that functions f and g and their derivatives with respect to x have the following values at $x = 2$ and $x = 3$.

Check the appropriate box for the derivatives with respect to x of the following combinations at the given value of x.

	$\frac{37}{6}$	$\frac{\sqrt{2}}{24}$	$15 - 8\pi$	$-\frac{5}{3\sqrt{17}}$	$\frac{5}{32}$	-1	other
$f(x) \cdot g(x)\,,\ x = 3$							
$f(x)/g(x)\,,\ x = 2$							
$f(g(x))\,,\ x = 2$							
$\sqrt{f(x)}\,,\ x = 2$							
$1/g^2(x)\,,\ x = 3$							
$\sqrt{f^2(x) + g^2(x)}\,,\ x = 2$							

9 points

4) Each of the following functions has exactly one critical point in the interval $[1, 2]$. Check the appropriate boxes regarding the value a of that critical point and its type.

	$a = 1$	$a = \sqrt{3}$	$a = 2$	$a = \frac{3}{2}$	$a = \sqrt{2}$	$a = \frac{\pi}{2}$	local min.	local max.	other
$f(x) = \dfrac{x^2 + x - 1}{x - 1}$									
$f(x) = 2x\sqrt{4 - x^2}$									
$f(x) = \pi x + \sin(\pi x)$									

4 points

5) Consider the function $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}$ and check the following statements.

	True	**False**
a. f is an odd function.	☐	☐
b. f has exactly one horizontal asymptote.	☐	☐
c. f has no inflection points.	☐	☐
d. f is increasing on the interval $(-\infty, \infty)$.	☐	☐

4 points

6) Consider the function $f(x) = x^{5/3} + 5x^{2/3}$ and check the following statements.

	True	**False**
a. f has no local minimum.	☐	☐
b. f has exactly one local maximum.	☐	☐
c. f is increasing for $x > 0$.	☐	☐
d. f is concave down for $x > 0$.	☐	☐

4 points

7) In order to find a positive root of the equation $x - 2\sin x = 0$, apply Newton's method with $x_0 = 2$ as a first approximation. Then the fourth approximation equals

$\boxed{a}$ 1.8737647 $\boxed{b}$ 1.8834261 $\boxed{c}$ 1.8954943 $\boxed{d}$ 1.9008769

$\boxed{e}$ None of the above.

6 points

8) A small rectangular sheet of metal 10 cm wide and 20 cm long is folded down the middle and triangles are glued to the ends to produce a v-shaped feed or water trough for a pet's cage. The angle of the bend to maximize the amount the trough can hold should be

$\boxed{a}$ $\frac{\pi}{4}$ $\boxed{b}$ $\frac{\pi}{3}$ $\boxed{c}$ $\frac{\pi}{2}$ $\boxed{d}$ $\frac{2\pi}{3}$

$\boxed{e}$ None of the above.

6 points

9) A cylindrical aluminum can which is capped on both ends is to hold 40 in^3 of juice. In order that the least material (i.e., aluminum) is used, the radius of the caps has to be (in inches)

$\boxed{a}$ $\left(\frac{10}{\pi}\right)^{1/3}$ $\boxed{b}$ $\left(\frac{20}{\pi}\right)^{1/3}$ $\boxed{c}$ $\left(\frac{40}{\pi}\right)^{1/3}$ $\boxed{d}$ $\left(\frac{80}{\pi}\right)^{1/3}$

$\boxed{e}$ None of the above.

8 points

10 a) Suppose that there is a differentiable function $y = f(x)$ such that x and y satisfy the equation $x^2y^2 - \frac{y^3}{x} = 66$ and such that $f(-4) = 2$. Then $f'(-4)$ equals

$\boxed{a}$ $\frac{1}{2}$ $\boxed{b}$ $\frac{31}{63}$ $\boxed{c}$ $\frac{45}{99}$ $\boxed{d}$ $\frac{63}{134}$

$\boxed{e}$ None of the above.

10 b) Suppose that there is a differentiable function $y = f(x)$ such that x and y satisfy the equation $y^3 + 2y - \sin(\pi x) - 3 = 0$ and such that $f(2) = 1$. Then $f'(2)$ equals

$\boxed{a}$ $\frac{\pi}{5}$ $\boxed{b}$ $\frac{2\pi}{5}$ $\boxed{c}$ $\frac{\pi}{3}$ $\boxed{d}$ $\frac{2\pi}{3}$

$\boxed{e}$ None of the above.

6 points

11) Two roads intersect at right angles. Two cars travel through the intersection simultaneously (barely missing a collision). The first car travels at a speed of 30 mph, traveling due north. The other car travels at a speed of 40 mph traveling due east. Thirty minutes later the distance between the cars is increasing at a rate of

[a] 50 mph [b] 55 mph [c] 60 mph [d] 65 mph

[e] None of the above.

4 points

12(a) Suppose that $F(x) = \int_1^{x^3-7x} \cos^4 t\, dt$. Then $F'(x)$ equals

[a] $(x^3-7x)\cos^4 x$ [b] $-4(x^3-7x)\cos^3 x \sin x$

[c] $(3x^2-7)\cos^4(x^3-7x)$ [d] $-4(3x^2-7)\cos^3 x \sin x$

[e] None of the above.

12(b) Suppose that $F(x) = \int_{\sin x}^{0} \sin(\cos t)\, dt$. Then $F'(x)$ equals

[a] $\sin^2(\cos x)$ [b] $-\cos(\sin(\cos x))$

[c] $-\sin(\cos(\sin x))$ [d] $(\cos x)[\sin(\cos(\sin x))]$

[e] None of the above.

4 points

13) Let $f(x) = \frac{1}{x}$ for $1 \le x \le 2$, and let $[1,2]$ be subdivided into 10 subintervals of equal length. Then the corresponding upper sum of f on $[1,2]$ to approximate $\int_1^2 \frac{1}{x}\, dx$ yields the value

[a] 0.7187714032 [b] 0.7188263197

[c] 0.7201385642 [d] 0.7221357686

[e] None of the above.

8 points

14) Check the appropriate box for the values of the following definite integrals:

	$\frac{\pi}{4}$	$\frac{19}{99}$	$\frac{1}{8}$	$\frac{1}{16}$	other
$\int_0^1 \frac{x^5}{(x^6+1)^3}dx$					
$\int_4^9 \frac{\sqrt{x}}{(30-x^{3/2})^2}dx$					
$\int_0^{\pi/4} \sin^3 2x \cos 2x\, dx$					
$\int_0^{\pi/2} \cos^2 x\, dx$					

5 points

15) The area of the region bounded by the graphs of $x = y^2$ and $x = 4 - y^2$ equals

$\boxed{a}$ $8\sqrt{2}$ $\boxed{b}$ $\frac{8}{3}\sqrt{2}$ $\boxed{c}$ $\frac{16}{3}\sqrt{2}$ $\boxed{d}$ $16\sqrt{2}$

$\boxed{e}$ None of the above.

5 points

16) A spike is six inches long and has cross sections which are equilateral triangles with sides of length $1/6$ the distance from the point of the spike (so the side length at the blunt end is exactly 1 inch). The volume of the spike equals (in inches)

$\boxed{a}$ $\frac{\sqrt{3}}{2}$ $\boxed{b}$ $\frac{3}{4}\sqrt{3}$ $\boxed{c}$ $\sqrt{3}$ $\boxed{d}$ $\frac{3}{2}\sqrt{3}$

$\boxed{e}$ None of the above.