Applied Mathematics Syllabus

1. Ordinary Differential Equations

    - Initial value problem – existence, uniqueness.
    - One-step methods for the numerical solution of the initial value problem - explicit and implicit schemes. Numerical solution of nonlinear equations.
    - Stability and phase plane analysis, bifurcations.
    - Applications such as population models, epidemiology.
    - Boundary value problems; finite difference methods - numerical solution of banded linear systems.
    - Sturm-Liouville systems.

2. Fourier series and orthogonal expansions. Discrete Fourier series and convolutions.

3. The heat equation.

    - Heat flow, Fick's law.
    - Separation of variables.
    - Fundamental solution from Fourier transforms; scale-invariance.
    - Smoothing effect, maximum principle.
    - Finite difference methods for heat equation in 1-3 dimensions.

4. Quasilinear first-order equations, characteristics, Burger's equation.

5. The wave equation.

    - 1D - d'Alembert's formula, initial-boundary value problems.
    - 2D, 3D: method of spherical means, Hadamard's method of descent.

- Inhomogeneous equations via Duhamel's principle.
- Domain of influence/dependence, Huygen's principle.
- Conservation of energy.
- Numerical issues.

6. Distributions and the delta function; Green's functions and fundamental solutions.
7. Laplace and Poisson equations.
   - Separation of variables for special geometries.
   - Green's representation for solution to Dirichlet problem, Poisson integral.
   - Mean value inequality, strong and weak maximum principles, uniqueness for Dirichlet problem
   - Dirichlet Principle
   - Issues in finite difference methods for Poisson equations.
   - A simple finite element method.
8. Elementary fluid dynamics.
   - Derivation of equations of motion.
   - Vortex dynamics.
   - Conformal mappings and fluid flow.
9. Dimensional analysis and scaling.
10. Perturbation theory for ODE's, asymptotic methods.
    - Regular perturbation.
    - Asymptotic series.

- Multiple scales, secular terms.
- Boundary layers, matching.
- Asymptotic methods for integrals; Stirling's formula.

11. Discrete models.

- Examples from population dynamics, economics.
- Linear difference equations.