PROOF OF FORMULA 3.522.10

\[
\int_0^\infty \frac{x \, dx}{(1 + x^2) \cosh \frac{\pi x}{4}} = \frac{1}{\sqrt{2}} \left( \pi - 2 \ln(\sqrt{2} + 1) \right)
\]

This is entry \textbf{3.522.3} with \(a = \pi/4\) and \(b = 1\). Therefore
\[
\int_0^\infty \frac{x \, dx}{(1 + x^2) \cosh \frac{\pi x}{4}} = 4 \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{4k - 1}.
\]

To evaluate the series integrate
\[
\sum_{k=1}^{\infty} (-1)^k x^{4k-2} = \frac{x^2}{1 + x^4}
\]
to obtain
\[
\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{4k - 1} = \int_0^1 \frac{x^2 \, dx}{1 + x^4}.
\]

The factorization
\[
1 + x^4 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)
\]
gives the integral by partial fractions.