PROOF OF FORMULA 4.215.3

\[ \int_0^1 \sqrt{\ln \frac{1}{x}} \, dx = \frac{\sqrt{\pi}}{2} \]

Let \( t = \ln \frac{1}{x} \) to produce

\[ \int_0^1 \left( \ln \frac{1}{x} \right)^{\mu-1} \, dx = \int_0^\infty t^{\mu-1} e^{-t} \, dt. \]

This is the standard integral representation of the gamma function, appearing as entry 8.310.1. Therefore

\[ \int_0^1 \left( \ln \frac{1}{x} \right)^{\mu-1} \, dx = \Gamma(\mu). \]

Now put \( \mu = \frac{3}{2} \) to obtain the result.