PROOF OF FORMULA 4.229.3

\[ \int_0^1 \ln(\ln 1/x) \frac{dx}{\sqrt{\ln 1/x}} = -(\gamma + 2 \ln 2) \sqrt{\pi} \]

Entry 4.229.4 states that

\[ \int_0^1 \ln(\ln 1/x) (\ln 1/x)^{\mu-1} \, dx = \psi(\mu) \Gamma(\mu) \]

The result now follows from the values \( \psi(1/2) = -(\gamma + 2 \ln 2) \) (given as entry 8.366.2 and \( \Gamma(1/2) = \sqrt{\pi} \).