

Calculus 121: Practice Test

- 1) Is $f(x) = (x^2 - 1)/(x - 1)$ a polynomial function? Give a detailed answer.
- 2) A line of length 10 is cut into two pieces. Make a square and a circle out of the two pieces. Find the sum of the two areas as a function of the place where you cut.
- 3) Find the equation of the line that passes through the point $(1, 3)$ and is perpendicular to the line $3x - 2y = 1$.
- 4) Let $f(x) = 3x + 7$. Find the inverse of f .
- 5) Let $h(x) = x^2 + 4x + 3$. Does h have an inverse? If yes, find it. If no, restrict the domain of h so that h is now invertible. Find the inverse of h explicitly.
- 6) Graph the function $t(x) = |2x - 3| + |5x - 2|$.
- 7) Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{25(5+h)^{-2} - 1}{h}.$$

- 8) Describe what happens to the roots of the quadratic equation $ax^2 + bx + c = 0$ as the coefficient $a \rightarrow 0$. You will have to compute two limits.
- 9) Prove that a polynomial of odd degree must have at least one root.
- 10) Prove that the function

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x < 4 \\ 24/(x - 3) & \text{if } x \geq 4 \end{cases}$$

is continuous at $x = 4$.

- 11) Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ a & \text{if } x = 0. \end{cases}$$

Find a value of a for which the function is continuous. Does the derivative of f at $x = 0$ exist?

- 12) Find the derivative of $f(x) = 1/\sqrt{x}$ at $x = 9$ by using the definition as a limit.
- 13) Prove by the definition that the derivative of $f(x) = \cos x$ is $f'(x) = -\sin x$.