Homework 1. Solutions

Problem 6. Remember that in life you might grow and shrink.

Problem 23. From
\[ C = \frac{5}{9}(F - 32) \]
you should get
\[ F = \frac{9}{5}C + 32. \]
If \( F \geq -459.67 \), then \( C \geq -273.15 \). Do you know what these numbers mean? If not, you should find out.

Problem 24. You should get
\[ v = \frac{c}{\sqrt{1 - \frac{m^2}{c^2}}} \]
Observe that as \( m \to \infty \) you get \( v \to c \). You should graph \( v = v(c) \) to interpret its meaning.

Problem 37. First note that \( f(x) = 9 - x^2 \), \( 0 \leq x \leq 3 \) corresponds to one of the branches of the parabola. To check that \( f \) is one-to-one, suppose \( f(x_1) = f(x_2) \). This gives \( 9 - x_1^2 = 9 - x_2^2 \) and from here \( x_1^2 = x_2^2 \). It now follows that \( x_1 = x_2 \) because they are both positive.

Inverting \( f \) we get \( g(x) = \sqrt{9-x} \). Then, without using Theorem 7 we have
\[ g'(x) = \frac{-1}{2\sqrt{9-x}} \]
so \( g'(8) = -1/2 \). To use Theorem 7, we compute \( g(8) = 1 \) and then \( g'(8) = 1/f'(1) \) and \( f'(x) = -2x \). The two results agree.

Problem 43. From Theorem 7 you know that
\[ g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(4)} = \frac{3}{2} \]

Problem 44. Use the quotient rule and proceed as in Problem 43.

Problem 50. To obtain the required expression, differentiate twice the relation \( g(f(x)) = x \).