Honors Calculus 131. Practice Test 1.
Due September 22.

1) Give a precise definition of the inverse function of $f(x) = \sin x$ by giving a proper domain and range. Prove that

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}.$$

2) Let $g(x) = 5x + \ln x$. Assume that g has an inverse function $h(x)$. Evaluate $h'(5)$.

3) Evaluate

$$\lim_{x\to 0} \frac{\tan 5x}{\tan 3x}$$

without using L'Hopital's rule.

4) Evaluate

$$\lim_{x\to 0} \frac{11^x - 3^x}{x}$$

without using L'Hopital's rule.

5) Evaluate

$$\lim_{x\to 2} \frac{\sqrt{2x+5}-3}{\sqrt{x+14}-4}$$

without using L'Hopital's rule.

6) Let

$$t(x) = \begin{cases} 4 + x^3 & \text{if } x < 1 \\ (2x+3)/(x^2+x-1) & \text{if } x > 1. \end{cases}$$

a) Does the limit of $t(x)$ as $x \to 1$ exist?
b) Is $t(x)$ continuous at $x = 1$? If not, can you modify it to make it so.
c) Is $t(x)$ differentiable at $x = 1$?

7) Determine what happens to the roots of the equation $ax^2 + bx + c = 0$ as $a \to 0$.

8) Use the definition of limit to prove that

$$\lim_{x\to 2} \frac{1}{x} = \frac{1}{2}.$$

9) Evaluate

$$\lim_{x\to+\infty} \sqrt{x^2+ax} - \sqrt{x^2+bx}$$

in terms of the parameters a and b.

10) Prove that

$$\lim_{x\to+\infty} f(x) = \lim_{x\to 0^+} f(1/x).$$

11) Find the derivative of $f(x) = x|x|$. Play special attention at $x = 0$.

12) Find the derivative of $f(x) = x^3 + 7x$ at $x = 1$ using the definition. Write the equation of the tangent line at $x = 1$.

13) A fixed point for a function f is an element $c \in \text{Domain}(f)$ such that $f(c) = c$. Find the fixed points of $f(x) = x(1-x)$. Prove that every continuous functions defined on the interval $[0, 1]$ must have at least one fixed point. **Hint**. Try to use the intermediate value theorem.

14) Suppose f is a function that satisfies the identity

$$f(x+y) = f(x) + f(y) + x^2y + xy^2$$

for all real numbers x and y. Suppose also that

$$\lim_{x\to 0} \frac{f(x)}{x} = 1.$$

a) Find $f(0)$.
b) Find $f'(0)$.
c) Find $f'(x)$.

15) Find equations of the lines that are tangent to the curve $y = 1 + x^3$ and are parallel to the line $12x - y = 1$.

16) How many lines passing through the point $(0, c)$ are normal to the curve $y = x^2$?

17) If $a(x) = f(x)g(x)$, find a formula for the second and third derivatives of a in terms of the derivatives of f and g. Make a guess for a general formula.

18) Compute the derivative of $f(x) = \tan x$ by definition and then check your result by using the quotient rule.

19) Suppose that f is a differentiable function and $a \in \mathbb{R}$. Compute the derivatives of $f(x^a)$ and $(f(x))^a$.

20) Use the chain rule and the product rule to prove the quotient rule for derivatives.

21) Use the definition of derivative to evaluate

$$\lim_{x\to 0} \frac{\ln(1+x)}{x}.$$

22) Suppose f is a differentiable function. Evaluate

$$\lim_{x\to a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}},$$

in terms of the function f and its derivatives.

23) Let $q(x) = x^a \sin(1/x^b)$ for $x \neq 0$. Here a and b are positive numbers. Define $q(0)$ to make q continuous. For what values of a and b is $q(x)$ differentiable at $x = 0$? Is the function $q'(x)$ continuous at $x = 0$? Find restrictions on a and b that guarantee the existence of $q''(0)$.

24) If f and g are differentiable functions at $x = 0$ with $f(0) = g(0) = 0$ and $g'(0) \neq 0$, prove that

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{f'(0)}{g'(0)}.$$