# Honors Calculus 131. Problem set 4.
# Due September 29.

1) If $a$ and $b$ are positive numbers, find the maximum value of $f(x) = x^a(1-x)^b$ for $0 \le x \le 1$.

2) A cubic polynomial is a function of the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \ne 0$. Count the number of critical points and give one example of each type.

3) Show that the equation $4x^5 + x^3 + 2x + 1 = 0$ has exactly one real root.

4) Prove that $\sqrt{1+x} < 1 + x/2$ for $x > 0$.

5) If $a_1 < a_2 < \cdots < a_n$, find the minimum value of the

$$f(x) = \sum_{i=1}^{n} (x - a_i)^2.$$

6) Let $a > 0$. Find the maximum value of the function

$$f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-a|}.$$

7) Prove that $\frac{1}{9} < \sqrt{66} - 8 < \frac{1}{8}$ without computing the square root.

8) Suppose $f$ is a function that satisfies $f'(x) = 1/x$ for all $x > 0$ and $f(1) = 0$. We will prove later that $f(x) = \ln x$. Prove that $f(xy) = f(x) + f(y)$ for all $x$, $y > 0$. **Hint**. Define the function $g$ by $g(x) = f(xy)$ with $y$ kept fixed. Now compute the derivative of $g$.