Math 307: Probability. Problems for Test 1

(1) Two cards are drawn successively from a deck of 52 cards. Find the probability that the second card is higher in rank than the first card.
(2) A coin is tossed until the first time that a head turns up. Find the probability that the coin turns up head on the tenth toss.
(3) A die is rolled 30 times. What is the probability that a 6 turns up exactly 5 times? What is the most probable number of times that a 6 will turn out?
(4) A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards. Find the probability of a full house (one pair and one triple). Find the probability of a straight (five cards in a row, aces count as 1 or the highest card).
(5) Prove the identity

$$\sum_{j=0}^{n} \binom{n}{j}^2 = \binom{2n}{n}.$$

Hint: consider an urn with n red balls and n brown balls. Show that each side counts the number of ways to choose $2n$ balls.
(6) A card is drawn at random from a deck of 52 cards. What is the probability that it is a heart, given that is red? What is the probability that it is a jack, given that is red?
(7) What is the probability that a family of two children has a) two boys given that it has at least one boy? b) two boys given that the first child is a boy?
(8) Two cards are drawn from a deck of 52 cards. What is the probability that the second drawn card is red?
(9) Prove that if A and B are independent events so are A and B complement.
(10) A die is thrown twice. Let X_1 abd X_2 denote the outcomes. Define $X = \min(X_1, x_2)$. Find the distribution of X.
(11) A die is rolled until the first time T that a six turns up. a) Find the probability distribution for T. b) Find $P(T > 3)$ and $P(T > 6|T > 3)$.
(12) A die is loaded in such a way that the probability that a 6 is thrown is five times that of any other number, each of which is equally probable. Throw the die four times. Compute the probability of obtaining at least one 6. Compare that with the result for a fair die.
(13) An urn contains b blue balls and y yellow balls. A ball is drawn at random, its color is recorded, and then it is returned to the urn together with d further balls of the same color. a) What is the probability that the third ball is blue? b) Let C_n denote the event that the nth ball is blue. Find $P(C_n)$.